

17EC54

## Fifth Semester B.E. Degree Examination, July/August 2021 Information Theory and Coding

Time: 3 hrs .
Max. Marks: 100

## Note: Answer any FIVE full questions.

1 a. A code is composed of dots and dashes. Assuming that a dash is 3 times as long as a dot and has one-third the probability of occurrence. Calculate:
(i) The information in a dot and a dash
(ii) The entropy of dot-dash code
(iii) The average rate of information if a dot lasts for 10 m -sec and this time is allowed between symbols.
(08 Marks)
b. A zero-memory source has a source alphabet. $\mathrm{S}=\left\{\mathrm{s}_{1}, \mathrm{~s}_{2}, \mathrm{~s}_{3}\right\}$ with $\mathrm{P}=\{1 / 2,1 / 4,1 / 4\}$. Find the entropy of this source and its $2^{\text {nd }}$ extension. Also verify that $H\left(s^{2}\right)=2 H(s)$.
(06 Marks)
c. Derive the expression to show that $\mathrm{n}^{\text {th }}$ extension entropy of the basic binary source $\mathrm{H}\left(\mathrm{s}^{\mathrm{n}}\right)=\mathrm{n} \mathrm{H}(\mathrm{s})$.
(06 Marks)
2 a. The state diagram of a Markoff source is shown in Fig.Q2(a):
(i) Find the entropy H of the source
(ii) Find $\mathrm{G}_{1}, \mathrm{G}_{2}$ and $\mathrm{G}_{3}$ and yerify that $\mathrm{G}_{1}>\mathrm{G}_{2}>\mathrm{G}_{3}>\mathrm{H}$


Fig.Q2(a)
(12 Marks)
b. Suppose that $s_{1}$ and $s_{2}$ are two zero memory sources with probabilities $p_{1}, p_{2} \ldots p_{n}$ for source $\mathrm{s}_{1}$ and $\mathrm{q}_{1}, \mathrm{q}_{2} \ldots \mathrm{q}_{\mathrm{n}}$ for source $\mathrm{s}_{2}$. Show that the entropy of source $\mathrm{s}_{1}$.

$$
\mathrm{H}\left(\mathrm{~s}_{1}\right) \leq \sum_{\mathrm{k}=1}^{\mathrm{n}} \mathrm{p}_{\mathrm{k}} \log \frac{1}{\mathrm{q}_{\mathrm{k}}}
$$

(08 Marks)

3 a. Explain properties of codes.
(08 Marks)
b. Apply Shanon's encoding algorithm to the following message
$\mathrm{S}=\mathrm{S}_{1} \quad \mathrm{~S}_{2} \quad \mathrm{~S}_{3}$
$\mathrm{P}=\begin{array}{llll}0.5 & 0.3 & 0.2\end{array}$

Find code efficiency and redundancy for the basic source and its $2^{\text {nd }}$ order extension source.
(12 Marks)
4 a. Construct a binary and ternary Huffman code for the source with 8 alphabets A to H with respective probabilities $0.22,0.20,0.18,0.15,0.10,0.08,0.05,0.02$. Determine efficiency for both the codes.
(12 Marks)
b. Explain:
(i) Arithmetic coding
(ii) Lempel-Ziv algorithm

5 a. Show that the mutual information of a channel is symmetric.
(08 Marks)
b. For the JPM given below, compute individually $\mathrm{H}(\mathrm{X}), \mathrm{H}(\mathrm{Y}), \mathrm{H}(\mathrm{X}, \mathrm{Y}), \mathrm{H}(\mathrm{X} / \mathrm{Y}), \mathrm{H}(\mathrm{Y} / \mathrm{X})$ and I(X, Y)

$$
\mathrm{P}(\mathrm{X}, \mathrm{Y})=\left[\begin{array}{cccc}
0.05 & 0 & 0.20 & 0.05 \\
0 & 0.10 & 0.10 & 0 \\
0 & 0 & 0.20 & 0.10 \\
0.05 & 0.05 & 0 & 0.10
\end{array}\right]
$$

(12 Marks)

6 a. Derive the expression of channel capacity for binary symmetric channel.
(08 Marks)
b. Find the channel capacity of the channel matrix shown using Murgoa's method. The data transmission rate is 10,000 symbols $/ \mathrm{sec}$.

$$
\mathrm{P}(\mathrm{Y} / \mathrm{X})=\left[\begin{array}{ccc}
0.8 & 0.2 & 0 \\
0.1 & 0.8 & 0.1 \\
0 & 0.2 & 0.8
\end{array}\right]
$$

(08 Marks)
c. Define the terms:
(i) PRIORIEntropy
(ii) Posteriori (conditional) entropy
(iii) Equiyocation
(iv) Mutual information
(04 Marks)
7 a. For a systematic $(7,4)$ linear block code, the parity check matrix P is given by

$$
[\mathrm{P}]=\left[\begin{array}{lll}
1 & 1 & 1 \\
1 & 1 & 0 \\
1 & 0 & 1 \\
0 & 1 & 1
\end{array}\right]
$$

(i) Find all possible code vectors.
(ii) Draw the encoder and syndrome calculation circuit.
(iii) Direct and correct the single errors in the received vector $\mathrm{R}_{\mathrm{A}}=[0111110]$ and $R_{B}=[1010000]$.
b. Design a single error correcting code with a message block size of 11 and show that by an example that it can correct single error.
(08 Marks)
8 a. For the $(7,4)$ single error correcting code $g(x)=1+x+x 3$. Find the code vector for the message vectors $\mathrm{D}=[1001]$ and $\mathrm{D}=[1101]$. Using systematic method. Also draw the encoder for $(7,4)$ cyclic code.
(10 Marks)
b. A $(15,5)$ linear cyclic code has a generator polynomial $g(x)=1+x+x^{2}+x^{4}+x^{5}+x^{8}+x^{10}$
(i) Draw the encoder and syndrome calculation circuit.
(ii) Find the code polynomial for $\mathrm{D}(\mathrm{x})=1+\mathrm{x}^{2}+\mathrm{x}^{4}$ using shift registers.
(iii) Is $\mathrm{V}(\mathrm{x})=1+\mathrm{x}^{4}+\mathrm{x}^{6}+\mathrm{x}^{8}+\mathrm{x}^{14}$ a code polynomial?
(10 Marks)
9 a. Consider the $(3,1,2)$ convolutional code with $g^{(1)}=(110), g^{(2)}=(101)$ and $g^{(3)}=(111)$.
(i) Draw the encoder block diagram.
(ii) Find the code word to the information sequence (11101) using time-domain and transform domain approach.
(10 Marks)
b. Write short notes on:
(i) Golay codes
(ii) BCH codes
(10 Marks)

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10 a. For the $(2,1,2)$ convolutional encoder $g^{(1)}=111, g^{(2)}=(101)$. Draw the encoder diagram Also write the state table, state transition table, state diagram and the corresponding code tree. Using the code tree, find the encoded sequence for the message (10111). Verify the output sequence so obtained using transform domain approach.
(14 Marks)
b. For the convolutional encoder shown in Fig.Q10(b), find the encoded sequence for the information sequence 10111 using both time domain and transform domain approach.


Fig.Q10(b)
(06 Marks)

